PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 02

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Today's Targets

1. Types of Relations
2. [redacted]
3. [redacted]
4. [redacted]

Homework

Let R be a relation defined as xRy if and only if $2x + 3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R?

A. 2

B. 3

C. 4

D. 6

x	y	R
1	$6 \in N$	(1,6)
2	$\frac{16}{3} \notin N$	×
3	$\frac{14}{3} \notin N$	×
4	$4 \in N$	(4,4)
5	$\frac{10}{3} \notin N$	×
6	$8/3 \notin N$	×
7	$2 \in N$	(7,2)
8	$4/3 \notin N$	×
9	$2/3 \notin N$	×
10	$0 \notin N$	×

$R = \{(1,6), (4,4), (7,2)\}$

Relation

Let A be the set of students of Class XII of a school and B be the set of students of Class XI of the same school. Then some of the examples of relations from A to B are

(i) $\{(a, b) \in A \times B : a \text{ is brother of } b\}$,

(ii) $\{(a, b) \in A \times B : a \text{ is sister of } b\}$,

(iii) $\{(a, b) \in A \times B : \text{age of } a \text{ is greater than age of } b\}$,

(iv) $\{(a, b) \in A \times B : \text{total marks obtained by } a \text{ in the final examination is less than the total marks obtained by } b \text{ in the final examination}\}$,

If $(a, b) \in R$, we say that a is related to b under the relation R and we write as aRb. In general, $(a, b) \in R$, we do not bother whether there is a recognizable connection or link between a and b. As seen in Class XI, functions are special kind of relations.

Types of Relation

Definition 1: A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

↳ Empty Relation

Definition 2: A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

↳ Universal relation

Question

Let A be the set of all students of a boys school. Show that the relation R in A given by $R = \{(a, b) : a \text{ is sister of } b\}$ is the empty relation and $R' = \{(a, b) : \text{the difference between heights of a and b is less than 3 meters}\}$ is the universal relation.

→ $R : A \to A$

h_a = height of a
h_b = height of b

$R = \{(a,b) : a \text{ is sister of } b\}$

a and b both are boys

∴ a can not be sister of b.

$R = \phi$

↳ Empty Relation

$R' = \{(a,b) : |h_a - h_b| < 3m\}$

Since, 300 cm ie. 3m is a large value w.r.t height of human being.

a is related to b for any $a, b \in A$

$R = A \times A$

↳ i.e. Universal Relation

Types of Relation

Definition 3: A relation R in a set A is called

(i) reflexive, if $(a, a) \in R$, for every $a \in A$,

(ii) symmetric, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

(iii) transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

Definition 4: A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Question

$A = \{1, 2, 3\}$

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1,1), (2,2), (3,3), (1,2), (2,3)\}$ is reflexive but neither symmetric nor transitive.

① Reflexive

for every $a \in A$

$\Rightarrow (a,a) \in R$

$a = 1, 2, 3 \in A$

$(1,1), (2,2), (3,3) \in R$

this R is reflexive

② Symmetric

for $a, b \in A$

and $(a,b) \in R$

$\Rightarrow (b,a) \in R$

But $(1,2) \in R$

$(2,1) \notin R$

Not symmetric.

③ Transitive

$a, b, c \in A$

and $(a,b), (b,c) \in R$

$\Rightarrow (a,c) \in R$

But $(1,2), (2,3) \in R$

$(1,3) \notin R$

Not transitive

Question

Let L be the set of all lines in a plane (2D) and R be the relation in L defined as $R = \{(L_1, L_2): L_1 \text{ is perpendicular to } L_2\}$. Show that R is symmetric but neither reflexive nor transitive.

① Reflexive

$L_1 \perp L_1$ (No)

$\Rightarrow (L_1, L_1) \notin R$

$\Rightarrow$ R is not reflexive

② Symmetry

If $(L_1, L_2) \in R$

$\Rightarrow L_1 \perp L_2$

$\Rightarrow L_2 \perp L_1$

$\Rightarrow (L_2, L_1) \in R$

R is symmetric.

③ Transitive

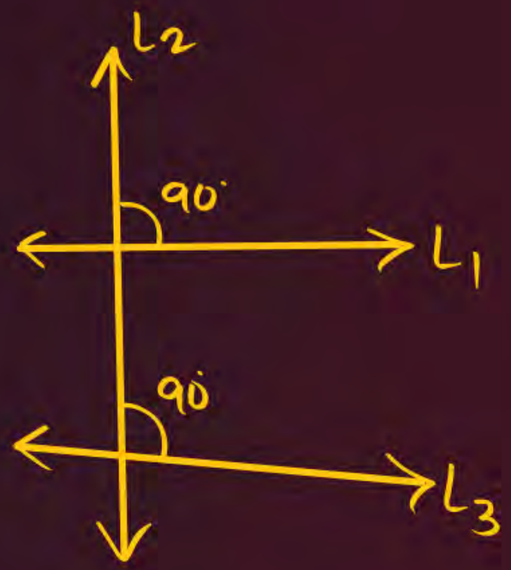
If $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow L_1 \perp L_2$, $L_2 \perp L_3$

$L_1 \perp L_3$ (No)

But $L_1 \parallel L_3$

Not transitive



Question

Show that the relation R in the set **Z** of integers given by

$$R = \{(a, b): 2 \text{ divides } a - b\}$$

is an equivalence relation.

① Ref.

$a - a = 0$

is divisible by 2

$\Rightarrow (a, a) \in R$

$\Rightarrow$ R is reflexive

② Symmetry

If $(a,b) \in R$

$\Rightarrow a - b = 2n$

$\Rightarrow b - a = -2n$

$(b-a)$ is also divisible by 2

$\Rightarrow (b,a) \in R$

$\Rightarrow$ R is symmetric

③ Transitive

If $a, b, c \in Z$

and $(a,b) \in R, (b,c) \in R$

$a - b = 2n \quad b - c = 2m$

$(a-b) + (b-c) = 2n + 2m$

$a - c = 2(m+n)$

$\Rightarrow a - c$ is divisible by 2

$\Rightarrow (a,c) \in R$

$\Rightarrow$ R is transitive

R is an equivalence Relation (Ans)

Question

Let R be the relation in the set {1,2,3,4} given by R = {(1,2), (2,2), (1,1), (4,4), (1,3), (3,3), (3,2)}. Choose the correct answer.

A. R is reflexive and symmetric but not transitive. ✗

(1,2) ∈ R but (2,1) ∉ R

B. R is reflexive and transitive but not symmetric. ✓

C. R is symmetric and transitive but not reflexive. ✗

D. R is an equivalence relation.

{(1,2), (2,2), (1,1), (4,4), (1,3), (3,2), (3,3)}

Question

Let R be the relation in the set **N** given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

A $(2,4) \in R$ ✗

B $(3,8) \in R$ ✗

C $(6,8) \in R$ ✓

D $(8,7) \in R$ ✗

a	b	R	
1	3 < 6	✗	
2	4 < 6	✗	
3	5 < 6	✗	
4	6 = 6	✗	
5	7 > 6	✓	(5,7)
6	8 > 6	✓	(6,8)
7	9 > 6	✓	(7,9)
8	10 > 6	✓	(8,10)
9	11 > 6	✓	(9,11)
10	12 > 6	✓	(10,12)
⋮	⋮	⋮	⋮

Rule 1 : $a, b \in N$

Rule 2 : $b = a + 2$

Rule 3 : $b > 6$

Question

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

$R = \{(1,2), (2,1)\}$

① $(1,1), (2,2), (3,3) \notin R$
$\Rightarrow$ Not reflexive

② for $(1,2) \in R \Rightarrow (2,1) \in R$
$\Rightarrow$ symm

③ for $(1,2), (2,1) \in R$
but $(1,1) \notin R$
$\Rightarrow$ Not transitive

Question

(2D)

Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

① Reflexive

$L_1 \parallel L_1$ (Yes)

$\Rightarrow (L_1, L_1) \in R$

$\Rightarrow$ It is reflexive

② Symmetry

If $L_1 \parallel L_2$

$\Rightarrow L_2 \parallel L_1$

$\Rightarrow (L_2, L_1) \in R$

$\Rightarrow$ It is symmetric

③ Transitive

If $(L_1, L_2) \in R$ $(L_2, L_3) \in R$

$\Rightarrow L_1 \parallel L_2$ and $L_2 \parallel L_3$

$\Rightarrow L_1 \parallel L_3$ (Yes)

$\Rightarrow (L_1, L_3) \in R$

$\Rightarrow$ It is transitive

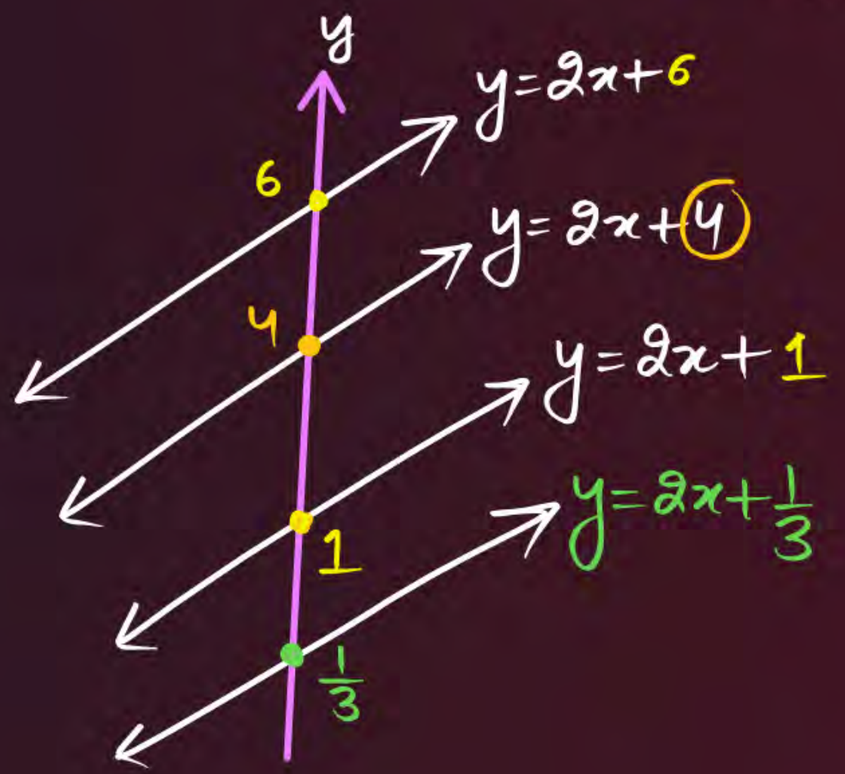
$\therefore$ R is an equivalence Relation.

The set of lines parallel to $y = 2x + 4$ is $\boxed{y = 2x + K}$ → koi bhi constant.

Slope - Intercept form

$y = mx + c$

m → Slope, c → y-intercept



Homework (HW)

Determine whether each of the following relations are reflexive, symmetric and transitive:

(iv) Relation R in the set **Z** of all integers defined as
$R = \{(x, y): x - y \text{ is an integer}\}$

(v) Relation R in the set A of human beings in a town at a particular time given by
(d) $R = \{(x, y): x \text{ is wife of } y\}$
(e) $R = \{(x, y): x \text{ is father of } y\}$

Solve (Ex 1.1) NCERT Class XII

THANK
YOU